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## **MODELING VAR OF DAX INDEX USING GARCH MODEL<sup>1</sup>**

### **Introduction**

This paper aims to answer question how much money would somebody need if he wanted insurance against very unusual decrease of German economy. To answer this question we help us a lit bit. Daily time series of German economy could be characterized by DAX Index. Very unusual situation let say once a 100 days. A tool or framework which we use to measure the potential loss will be concept of Value at Risk and crucial parameter we are going to play with is variance especially with assumption of non-constant variance of DAX Index.

The worthiness of the paper is to provide study how use dynamic model of variance in practical finance management. It is worth mention that the most important decision is on senior executives who have limited time and knowledge to make a decision what explained use of model with limited number of parameters.

### **1 FINANCIAL SERIES**

Financial time series can be characterized by two separate components – trend and fluctuations around components. The trend is mostly the strategic fundamentals for a given variable time series especially from a longer time perspective. Fluctuations rate in financial variables is called volatility and usually is measured by the square root of the variance.

In this paper we analyze The German Stock Index that is a total return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation. The DAX has a base value of 1,000 as of December 31, 1987. As of June 18, 1999 only XETRA equity prices are used to calculate all DAX indices

#### **1.1 Volatility of financial time series**

Typically the volatility has these features:

- **Volatility clustering:** in yield occur frequently phenomena that high volatility is followed by high volatility and low by low volatility, thus the volatility has the

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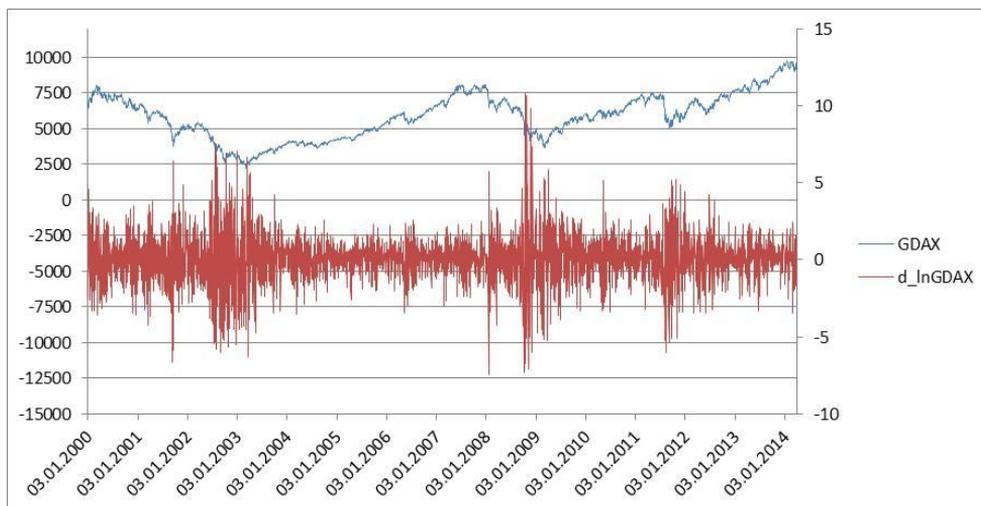
<sup>1</sup> This work was supported by the Grant Agency of Slovak Republic - VEGA, grant no. 1/0285/14 "Regional modelling of the economic growth of EU countries with concentration on spatial econometric methods"

autocorrelation characteristics. That is why it is interesting to use GARCH model for modeling the distribution of income, even when the model cannot explain this phenomenon.

- **Leverage effect:** refers to the well-established relationship between stock returns and both implied and realized volatility: volatility increases when the stock price falls. A standard explanation ties the phenomenon to the effect a change in market valuation of a firm's equity has on the degree of leverage in its capital structure, with an increase in leverage producing an increase in stock volatility.
- Volatility is **evolving continuously** over time, volatility jumps are continuous.
- Volatility **not diverges to infinity**, but in the long term is often stationary.

Graphical representation of DAX returns in figure 1 outlines all characteristics of volatility mentioned in previous paragraph:

Figure 1: Price [GDAX] and return [d\_InGDAX] of DAX index



**GDAX:** price of the DAX index series

**d\_InGDAX:** continuously compounded returns series calculated as difference of logarithms of DAX index prices [later we will call it DAX return]

The figure 1 shows us that around 2002 (the Internet bubble bursting), 2008 (Global Financial Crisis - The active phase of the crisis, which manifested as a liquidity crisis) and 2012 (fears of contagion of the European sovereign debt crisis to Spain and Italy) occurred high increase of volatility of returns while between the years returns are less volatile. It proves empirical hypothesis that once volatility increases than stay high for some period and also when price fall down volatility increases. Last two features are visible as well while around 2008 in the worst financial crises ever the volatility was huge but still far away from infinity or with structural jumps.

After empirical describing of the figure 1 would be great somehow describe behavior of returns by model. It seems good idea to use Box – Jenkins methodology as a feasible model building procedure for the general class of autoregressive-integrated-moving average processes. They are called ARIMA models were developed by Box and Jenkins (1976) and they are described in next chapter.

## 1.2 Analysis of univariate time series

### 1.2.1 Standard time series models:

The class of ARIMA models is the most widely used for the prediction of second-order stationary processes<sup>2</sup>. It uses an iterative six-stage scheme summarized by Francq, Zakoian (2010):

- (i) a priori identification of the differentiation order  $d$  (or choice of another transformation);
- (ii) a priori identification of the orders  $p$  and  $q$ ;
- (iii) estimation of the parameters
- (iv) validation;
- (v) choice of a model;
- (vi) prediction

In general we use 1<sup>st</sup> order differentiation and for financial time series usually difference of logarithms. This transformation has an economic interpretation as continuously compounded returns of original financial time series.

ARMA( $p,q$ ) model for transformed economic variable  $y$  is written by equation:

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \dots + \varphi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

$y_t$  – observation of stochastic process (in fact time series variable) in time  $t$

$\varphi_0, \dots, \varphi_p, \theta_1, \dots, \theta_q$  – parameters or weights

$\varepsilon_t: t = 0, 1, \dots$  - is a sequence of uncorrelated random variable from fixed distribution (often assumed to be normal) with mean equals 0, constant variance, and covariance  $cov(\varepsilon_t, \varepsilon_{t-k}) = 0$  for all  $k \neq 0$

Underlying assumption for process modeling by ARMA model is that stochastic process  $y_t$  has to be stationary. A stochastic process is said to be strictly stationary if its properties are not affected by a change of time origin. More practical  $y_t$  has to be weak stationary what means that at least first- (e.g. mean) and second-order (e.g. variance) moments of stochastic process are unaffected by a change of origin. The stationary assumption implies that the mean and the variance of the process are constant. As we have seen, the variance of DAX return is not constant so the model is not appropriate to model

<sup>2</sup> To simplify presentation, we do not consider seasonal series, for which SARIMA models can be considered This methodology is proposed by Box et al. (2008), 4<sup>th</sup> edition of famous Box, Jenkins (1994)

or predict this time series. It seems that mean is constant near 0 what is consistent with theory no free lunch so average returns is near risk free interest rate or for many authors just equals 0.

### 1.2.2 GARCH – Generalized AutoRegressive Conditional Heteroskedasticity

Modeling financial time series is crucial even we transform non-stationary price time series into series of return which seems to be stationary in terms of mean but not in variance. To answer task how much money we need to protect against unusual decrease of DAX return conditional variance is more important than conditional mean.

In this chapter we show how to improve model to deal with time varying conditional variance. Major improvement is to incorporate heteroskedasticity model in time series analysis. The framework was proposed by Engle (1982) who suggested model for variance explained by lagged disturbances. Assuming conditional normality, a general specification of the evolution of  $y_t$  would be:

$$y_t | Y_{t-1}, X_t \sim N(g_t, h_t)$$

Where  $Y_{t-1} = \{y_{t-s}, s \geq 1\}$  and  $X_t = \{x_{t-s}, s \geq 0\}$  are realization of time series variables and where both  $g_t$  and  $h_t$  are functions of the variables in  $Y_{t-1}$  and  $X_t$ . Engle prefers to consider model with

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2; \varepsilon_t = y_t - g_t \text{ usually called model for errors}$$

It could sound difficult so back in reality we show specific model from ARCH<sup>3</sup> family. The AR(1) process for  $y_t$  combining with ARCH(1) errors

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t | E_{t-1}) = g_t = 0 \text{ - conditional mean of } \varepsilon_t$$

$$V(\varepsilon_t | E_{t-1}) = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \text{ - conditional variance of } \varepsilon_t$$

ARCH process is still rather for academic than for practice but sometimes especially in risk management of financial institution is used. Honestly model for practice need to be simple because any of parameters have to have economic explanation and the model must be acceptable by management so must be credible and sustainable. Also according pure statistical reason higher q in ARCH(q) model increases chance of negative variance.

As we mentioned before volatility measured by conditional variance of financial variable signaling volatility clustering in other words after high volatility occurs high volatility. Bollerslev (1986) observed that in many application of ARCH(q) relatively high

<sup>3</sup> ARCH means autoregressive conditional heteroskedasticity

q are required and to avoid problems with negative variance estimates he imposed lag structure of variance into  $h_t$  function. We propose only the AR(1) process for  $y_t$  combining with GARCH(1,1) to keep focus on usable model:

$$y_t = \varphi_0 + \varphi_1 y_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t | E_{t-1}) = g_t = 0 \text{ - conditional mean of } \varepsilon_t$$

$$V(\varepsilon_t | E_{t-1}) = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \text{ - conditional variance of } \varepsilon_t$$

It is worth to say that implicitly we assume distribution for  $\varepsilon_t$  is defined by first- and second-order moments (e.g normal or students). Parameters  $\varphi_1$  is less than 1 and higher than -1 to avoid divergence and  $\alpha_1, \beta_1$  are less than 1 and positive to avoid divergence and negative estimate of variance as well.

## 2 APPLICATION GARCH MODEL IN RISK MANAGEMENT

The recent financial crisis and its impact on the broader economy underscore the importance of financial risk management in today's world. At the same time, financial products and investment strategies are becoming increasingly complex. Today, it is more important than ever that risk managers possess a sound understanding of mathematics and statistics in order to ensure that the business model has fewer surprises. Volatility is a measure which is by definition about variability in general and applied to return series of financial instrument it is about uncertainty of future profits. GARCH models led to a fundamental change to the approaches used in finance, through an efficient modeling of volatility (or variability) of the prices of financial assets.

### 2.1 Specify and estimate Conditional Mean and Variance Models for DAX returns

To estimate a composite conditional mean and variance model for DAX returns we use mentioned composite model AR(1) for DAX return and GARCH (1,1) for variance of the return. We use software Matlab and its statistical toolbox by following algorithm from Matlab's tutorial<sup>4</sup>

Graphical representation of DAX index and its return are in figure 1 and volatility outlines all characteristics of volatility mentioned in chapter 1.1. In this chapter we try to approve that model we have chosen is accurate and suitable.

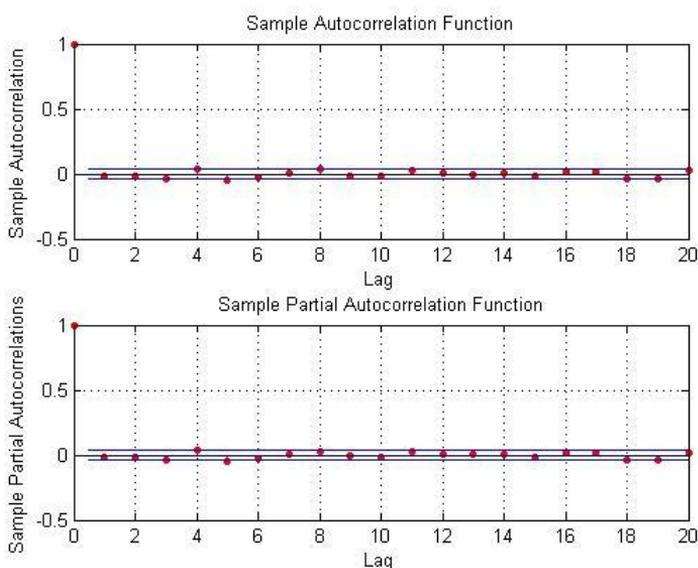
**Step 1. Load the data and specify the model:** Data was prepared from <http://finance.yahoo.com/q/hp?s=GDXI+Historical+Prices> and contained price time series of DAX from 3.1.2000 to 28.3.2014 which was transformed to the continuously compounded returns series by transformation:

<sup>4</sup> <http://www.mathworks.com/help/econ/conditional-mean-and-variance-model-for-nasdaq-returns.html#zmw57dd0e26313>

$$d_{\ln GDAX_t} = \ln \frac{GDAX_t}{GDAX_{t-1}} = \ln GDAX_t - \ln GDAX_{t-1}$$

**Step 2.** A priori **identification** of the orders  $p$  and  $q$  of **ARMA(p,q) model** of DAX return and validate hypothesis. We calculated autocorrelation and partial autocorrelation function and system calculated approximate 95% confidence bounds that parameter of lagged variable is 0. Figure 2 does not suggested significant AR (PACF) or MA (ACF) process due all estimation (red point) are within boundaries of confidence interval that parameter is 0 (lines very close to horizontal ax). Expertly we assume that there are AR(1) process so we test hypothesis 0 correlation.

Figure 2: ACF and PACF of returns time series



To ensure us of our hypothesis we conduct a Ljung-Box Q-test at lag 5. The null hypothesis for this test is that the first 5 autocorrelations are jointly zero and the testing statistics is  $\chi^2_5$

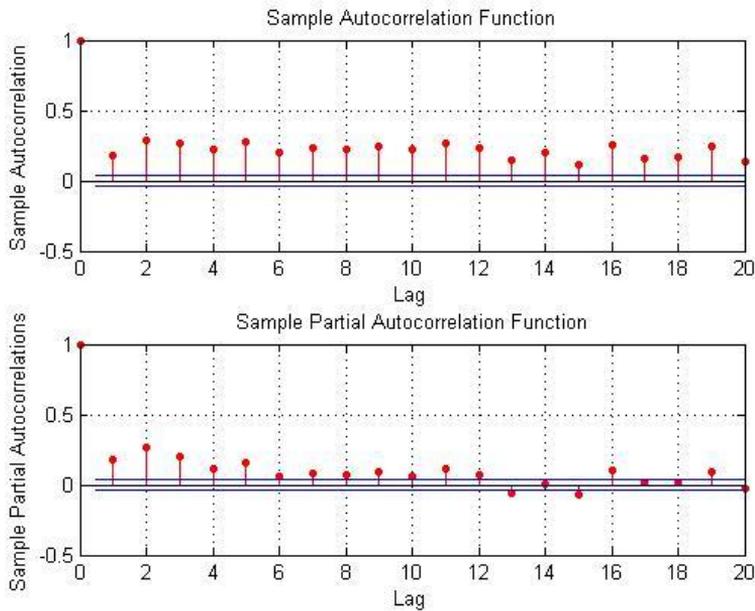
$$H_0: \rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho_5 = 0$$

$$H_1: \exists i; i \in \{1, \dots, 5\}: \rho_i \neq 0$$

p-value of test equals 0.00056 it is less than required significance level so we rejected null hypothesis and assume AR(1) process for DAX return.

**Step 3.** Check the series for **conditionalheteroskedasticity** and **test** for significant **ARChEffects**: Figure 3 shows the sample ACF and PACF of the squared DAX return series. The autocorrelation functions show significant serial dependence what indicates that the DAX return is conditionally heteroscedastic.

Figure 3: ACF and PACF of squared returns time series



To prove our suggestions we conduct an Engle's ARCH test that is a Lagrange multiplier test to assess the significance of ARCH effects. Our expert opinion is that GARCH(1,1) could be sufficient and was proved by Bollerslev that GARCH(P,Q) model is locally equivalent to an ARCH(P + Q) model so we conduct an Engle's ARCH test with two lags:

$$H_0: \alpha_0 = \alpha_1 = \alpha_2 = 0$$

$$H_1: \varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$$

p-value of test equals 0.0000 it is less than required significance level so we rejected null hypothesis and assume GARCH(1,1) process for DAX return variance.

**Step 4. Specify a conditional mean and variance model:** In Table 1 is specified parameters of the AR(1) model for the conditional mean of the DAX returns, and the GARCH(1,1) model for the conditional variance. This is a model of the form:

$$r_t = \varphi_0 + \varphi_1 r_{t-1} + \varepsilon_t \text{ [AR(1) model]}$$

where

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \text{ [GARCH(1,1) model]}$$

$z_t$  - is an independent and identically distributed standardized Gaussian process.

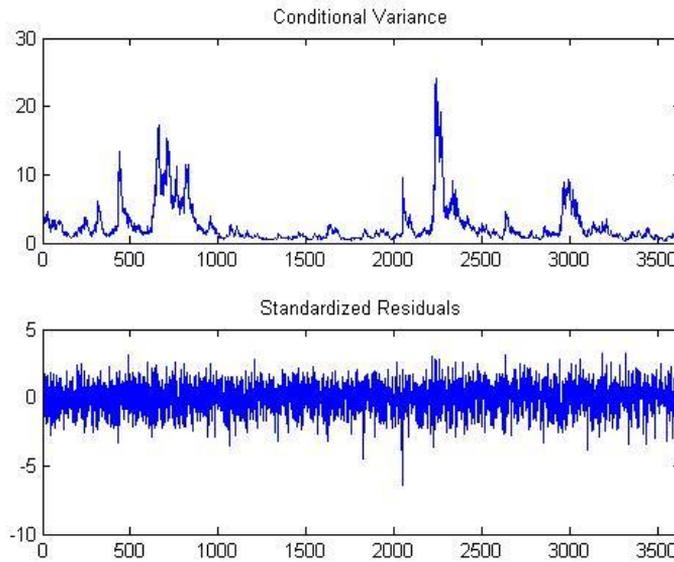
Table 1: Parameters of AR(1) for mean and GARCH(1,1) for variance

Model	Parameter	Value	Standard Error	t Statistic
AR(1)	Constant	0.0727	0.0185	3.9219
	AR{1}	-0.0249	0.0188	-1.3270
Model	Parameter	Value	Standard Error	t Statistic
GARCH(1,1)	Constant	0.0234	0.0040	5.9328
	GARCH{1}	0.9018	0.0079	114.3140
	ARCH{1}	0.0881	0.0073	12.1275

**Step 5. Infer the conditional variances and residuals:** Figures 4 shows that the conditional variances increase after observation 750, 2 250, 3 000. This corresponds to the increased volatility seen in the original return series at the end of 2002 - the Internet bubble bursting; at the end of 2008 – Global Financial Crisis (The active phase of the crisis, which manifested as a liquidity crisis); 2011 - fears of contagion of the European sovereign debt crisis to Spain and Italy

From part of Figure 4 we may conclude that the standardized residuals have more large values (larger than 2 or 3 in absolute value) than expected under a standard normal distribution. This suggests a Student's t distribution might be more appropriate for the innovation distribution. We conduct Jarque-Bera test normality where the null hypothesis is that the data of residuals comes from a normal distribution against alternative it does not. P value of test is 0.012 we rejected null hypothesis that data of residuals comes from a normal distribution. In next step improve model with assumption that data of residuals comes from Student's t distribution.

Figure 4: Conditional variance and standard residuals inferred from AR(1)/GARCH(1,1) model with Gaussian residual



**Step 6. Fit a model with an innovation from Student's  $t$  distribution:** In Table 2 is specified parameters of the AR(1) model for the conditional mean of the DAX returns, and the GARCH(1,1) model for the conditional variance. This is a model of the form

$$r_t = \varphi_0 + \varphi_1 r_{t-1} + \varepsilon_t \text{ [AR(1) model]}$$

where

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \text{ [GARCH(1,1) model]}$$

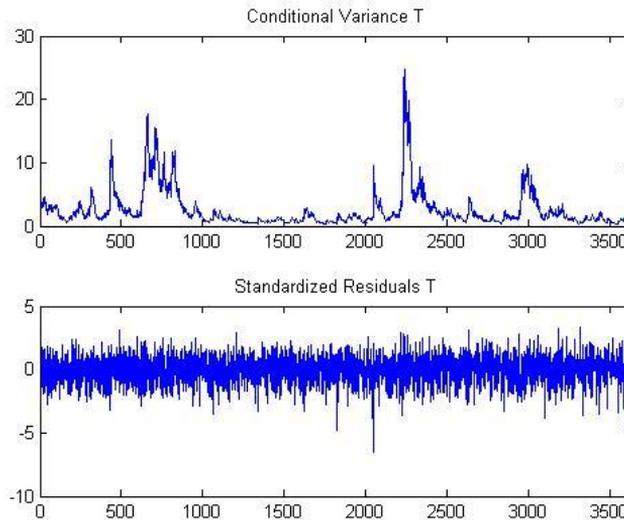
$z_t$  - is an independent and identically distributed Student's  $t$  process.

Table 2: Parameters of AR(1) for mean and GARCH(1,1) for variance

Model	Parameter	Value	Standard Error	t Statistic
AR(1)	Constant	0.0835	0.0181	4.6149
	AR{1}	-0.0207	0.0183	-1.1317
	DoF	10.1974	1.5734	6.4810
Model	Parameter	Value	Standard Error	t Statistic
GARCH(1,1)	Constant	0.0166	0.0046	3.5727
	GARCH{1}	0.9079	0.0095	95.0870
	ARCH{1}	0.0871	0.0095	9.1931
	DoF	10.1974	1.5734	6.4810

Graph of Conditional variance and standard residuals of this model in Figure 5 follow the same path as for normal distributed innovations.

Figure 5: Conditional variance and standard residuals inferred from AR(1)/GARCH(1,1) model with Student's residual



**Step 7. Compare the model fits:** Table 3 compares the two model fits (Gaussian and t innovation distribution) using the Akaike information criterion (AIC) and Bayesian information criterion (BIC).

Table 3: AIC and BIC criterion for suggested models

Stat	Conditional Probability Distribution:	
	Gaussian	t
AIC	12 217	12 162
BIC	12 248	12 199

The second model has six parameters compared to five in the first model (because of the t distribution degrees of freedom). Despite this, both information criteria favor the model with the Student's t distribution. The AIC and BIC values are smaller for the t innovation distribution

In previous paragraphs we prepared solid arguments to model DAX returns with composite AR(1) and GARCH(1,1) models (either with Gaussian or Student's innovation process) but we still have not answered the question or objective of this paper: how much money we need to insurance ourselves against unusual losses in German economy measured by daily losses of DAX Index. However we are close, the mentioned models allow quantitative measures of risk and uncertainty to be calculated by using time varying conditional variances. The next chapter is primarily devoted these calculations that are covered by well-known framework in financial industry Value at Risk [VaR].

## 2.2 Value at Risk

### 2.2.1. Definition and computing of Value at risk

Value at Risk is the most widely used risk measure in financial industry. In 1993, the business bank JP Morgan publicized its estimation method, RiskMetrics, for the VaR of a portfolio. VaR is now an indispensable tool for banks, regulators and portfolio managers. Hundreds of academic and nonacademic papers on VaR may be found at <http://www.gloriamundi.org/> also a lot of books were written about VaR, some of them became bestsellers i.e. Value at risk – the new benchmark for managing financial risk by Jorion (1996) provided the first comprehensive description of value at risk. It quickly established itself as an indispensable reference on VaR and has been called 'Industry standard'. Value at Risk theory and practice by Holton (2003) offers almost pure academic approach with extensive theory of risk measure and metric. Measuring market Risk by Dowd (2005) overviewed of the state of the art in market risk measurement. VaR summarizes the worst loss over a target horizon with a given level of confidence. Mathematical definition: if  $L$  is the loss of a portfolio, then  $VaR_{\alpha}(L)$  is the level  $\alpha$ -quantile, i.e.

$$VaR_{\alpha}(L) = \inf \{ l \in \mathbb{R} : P(L > l) = (1 - \alpha) \}$$

For practical demonstration is used 1 day 99% VaR of DAX index

$$VaR_{0.99}(L) = \inf \{ l \in \mathbb{R} : P(L > l) = 0.01 \}$$

With lower level of abstraction we may say that  $\text{VaR}_{0.99}$  is 99% percentil of daily loss distribution of DAX return.

Many papers could be found with the keywords calculation of VaR. Many of them may be at <http://gloria-mundi.com>. Basic classification by Dowd (2007):

- **Non-parametric approaches:**
  - Basic historical simulation
  - Bootstrapped historical simulation
  - Historical simulation using non-parametric density estimation
- **Parametric approaches:**
  - Unconditional distribution
  - Conditional distribution
- **Monte Carlo simulation**

### 2.2.2. Compute VaR using GARCH(1,1) and compare with other methods

VaR concept directly answer the question how much we need to be secure against 99% of losses<sup>5</sup> We have chosen from different classes of model for calculation VaR to show particular accuracy and sufficient of composite model developed in previous Chapter. To calculate is used floating window of 2 000 historical observation of DAX index daily return in fact loss.

- **Non-parametric approaches:**
  - Basic historical simulation [ $\text{VaR}_{99\_HS}$ ]
- **Parametric approaches:**
  - Unconditional distribution [ $\text{VaR}_{99\_ND}$ ;  $\text{VaR}_{99\_AR1}$ ]
  - Conditional distribution [ $\text{VaR}_{99\_G11\_nd}$ ;  $\text{VaR}_{99\_G11\_sd}$ ]

#### $\text{VaR}_{99\_HS}$

Historical simulation is based on empirical distribution. We do not need any special assumptions just order losses & profits from the worst loss to the best profit and 1<sup>st</sup> percentile is the value of insurance that assure us against 99% losses. For purposes of our analysis we use floating window of 2 000 observations as our empirical distribution what implies that the 20<sup>th</sup> worst loss is the number that we are looking for. This method is very conservative and could have very long memory based on observation period. Historical simulation is the most used method especially with time weights of observations to eliminate very out of date observations of loss.

#### $\text{VaR}_{99\_ND}$

This method of calculation is very simple while we assume that distribution of losses is normal. We need just estimate mean and variance from 2 000 observation then

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<sup>5</sup> By the word losses we mean complete distribution of profit and loss

VaR is calculated as 99<sup>th</sup> percentile of normal distribution with estimated mean and variance by the formula:

$$\mathbf{VaR99\_ND} = \mu - \mathbf{NormInv}(0.99) * \sigma$$

$\mu$                     sample mean  
 $\sigma$                     corrected sample standard deviation  
 NormInv                computes the inverse of the standard normal cdf

This class of method is rarely used due distribution of loss does not follow normal distribution and by using normal distribution usually the extreme losses are underestimate but the method is very simple and easy to calculate

### **VaR99\_AR1**

Almost the same method as VaR99\_ND however we assume that mean follows AR(1) process and therefore the model is a lit bit more dynamic. VaR is Calculated as 99<sup>th</sup> percentile of normal distribution with unconditional mean from AR(1) process and estimated variance what is described by formulas:

$$\mathbf{est}(d\_lnGDAX)_t = \varphi_0 + \varphi_1 * d\_lnGDAX_{t-1}$$

$$\mathbf{VaR99\_AR1} = \mathbf{est}(d\_lnGDAX)_t - \mathbf{NormInv}(0.99) * \sigma$$

$\mathbf{est}(d\_lnGDAX)$         forecast of profit/loss generated by AR(1) model  
 $\varphi_0, \varphi_1$                 parameters of AR(1) process  
 NormInv                computes the inverse of the standard normal cdf

### **VaR99\_G11\_nd**

Finally we can introduce VaR calculation by using model defined by the Table 1 from the chapter 2.1. All parameters seem to be consistent and logical. Parameter of AR(1) process is close to zero that mean of DAX return converges to constant and more important parameters of GARCH(1,1) are positive and less than 1 what means that every time estimate of variance is positive and variance converges to the constant however very slow according parameter  $\beta$  which is very close to 1 and we called it variance has very strong memory. Particular calculation of VaR is very simple like in the method VaR99\_ND but the estimation of mean and variance are obtained by composite model AR(1) (mean) and GARCH (1,1) (variance) defined by table 1 in chapter 2.1.

$$\mathbf{VaR99\_G11\_nd} = \mu - \mathbf{NormInv}(0.99) * \sqrt{\sigma^2}$$

$\mu$                     mean estimated by AR(1) model from table 2 of chapter 2.1  
 $\sigma^2$                 variance estimated by GARCH(1,1) model from table 2 of chapter 2.1  
 NormInv                computes the inverse of the standard normal cdf

**VaR99\_G11\_sd**

Important features of the model VaR99\_G11\_sd is said by previous paragraph however model is set by the table 2 in chapter 2.1. Even parameters from table 2 are very close to the table 1 however the estimated degrees of freedom are relatively small (about 10), indicating significant departure from normality. The calculation slightly change assuming Student's innovation process

$$VaR99_{ND} = \mu - TInv(0.99) * \frac{\sqrt{\sigma^2}}{\sqrt{\frac{df}{df-2}}}$$

$\mu$	mean estimated by AR(1) model from table 2 of chapter 2.1
$\sigma^2$	variance estimated by GARCH(1,1) model from table 2 of chapter 2.1
TInv	Student's t inverse cumulative distribution function
df	degree of freedom

These calculations of VaR methods is used to estimate 1 day 99% VaR using 2 000 historical observation each day working day from the 12th of November 2007 to the 28th of March 2014, it is 1 631 working days. To demonstrate accuracy of each method is calculated number of situation when estimated VaR is more then realized loss; it is called bridge.

Results of estimation VaR are shown by figure 6, figure 7 and figure 8. For all methods situation when realized loss was over VaR is correlated with failure of Lehman Brothers in September of 2008 and European sovereign debt crisis before all EU countries agreed to expand the EFSF by creating certificates that could guarantee up to 30% of new issues from troubled euro-area governments. Table 4 provide basic view of accuracy each method. From 1 631 estimation VaR by historical simulation only 18 times occurred situation when real loss was higher. But difference between VaR and time series of gain and loss is significant deep and this method should be noticed many banker as conservative. Unconditional parametric method are weak in terms of number bridges and difference is close to historical simulation. Path of profit/loss and GARCH(1,1) VaR time series shows correlation and by improvement with student distribution number of bridges are closer to theoretical expected values 16.3.

Figure 6: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by non-parametric method - historical simulation

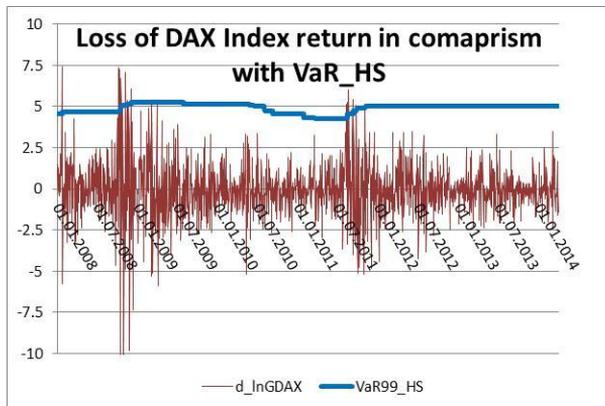


Figure 7: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by unconditional parametric methods – normal distribution, AR1 process

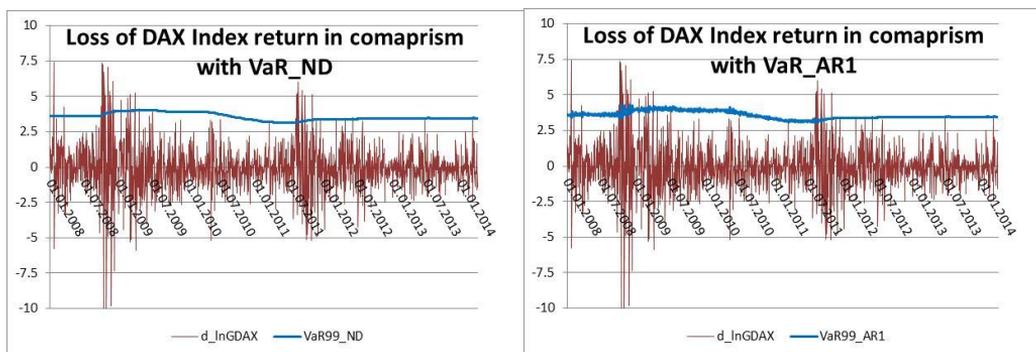


Figure 8: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by conditional parametric methods – GARCH (1,1) process

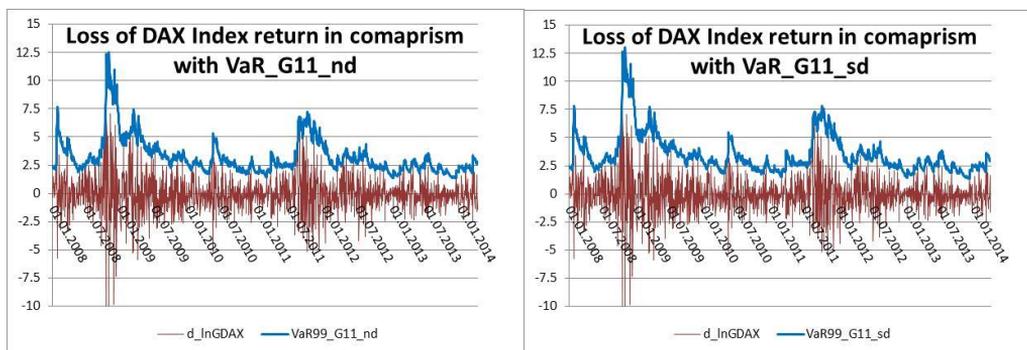


Table 4: Count situation when realized loss was above estimated VaR

VaR Method	No of bridge
VaR99_HS	18.0
VaR99_ND	38.0
VaR99_AR1	38.0
VaR99_G11_nd	33.0
VaR99_G11_sd	26.0
1% of 1631	16.3

## Conclusion

The subject of the article is to analyze the necessity of adopting conditional volatility model for returns as shows figure 1. Paper provided the basic demonstrations of theoretical result and illustrated the main techniques with numerical examples. Figure 4 and 5 proof that GARCH(1,1) is robust enough to model conditional volatility despite the fact that residual does not follow Gaussian distribution. Student's t distribution of innovation improves model slightly. GARCH (1,1) is easy to calculate using Matlab, correct volatility on average, exaggerates volatility-of-volatility. Example of calculating VaR using GARCH(1,1) shows a substantial gain in accuracy. VaR by GARCH(1,1) estimated number of situation when observed loss is higher than estimated VaR worse than historical simulation however path of this estimation is much closer to real observation.

## Keywords

DAX Index, volatility, GARCH, Value at Risk

## JEL Classification

C 44

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## RESUMÉ

Metodológia analýzy časových radov podľa Box-Jenkins je založená na predpoklade stacionarity a predpoklade, že rezídua ARMA modelunasledujú biely šum. Tieto predpoklady však nie sú často krát splnené pri analýze, modelovaní finančných časových radov. Táto práca približuje základne charakteristiky volatility finančných časových radov a prináša prehľad jedného z najpoužívanejších modelov na štatistický opis časového radu. Charakteristika volatility, ako aj špecifikácia kompozitného modelu podmienenej strednej hodnoty AR(1) a podmienenej variancie GARCH(1,1), sú demonštrované na časovom rade DAX indexu. Na koniec je metodológia GARCH aplikovaná na odhad VaR a konfrontovaná s ostatnými bežnými metódami výpočtu VaR.

## SUMMARY

The Box-Jenkins time series analysis rests on important concept as stationary and residuals of ARMA models follows white noise. These concepts are insufficient for the analysis of financial time series. The paper proposes main characteristic of volatility in financial time series and general overview of most common models for time series modeling. This paper also outlines characteristics of DAX index's volatility and shows how to specify a composite conditional mean and variance model using GARCH(1,1) model. We finally apply GARCH methodology to estimate VaR and compare with other approach.

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